HISTORY RETURNS:
INTERGENERATIONAL
MOBILITY OF EDUCATION
IN CHINA IN 1930–2010

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Introduction

During the twentieth century, China witnessed a dramatic episode of societal change when a series of revolutions toppled the social order that sustained the imperial rule for more than a thousand years. One of the most significant changes was the cessation of keju – the royal examination system that provided the only channel for upward social mobility – which permanently changed the nature of Chinese education. Education was no longer a bridge to enter officialdom, but a means to enhance one’s productivity and life satisfaction. The spread of education, however, was slow until the communist revolution of 1949. The Chinese Communist Party (CCP) began massive campaigns to elevate the Chinese population’s level of education. As a result, the number of illiterate or semi-illiterate people declined from 70 per cent among people born in the 1930s to about 10 per cent among those born around 1980.¹ The period of the Cultural Revolution (1966–76) saw the fastest growth in elementary and high school education, from which young

¹ Unless otherwise stated, figures reported in this section are from the authors’ calculations based on data from China Family Panel Studies (CFPS) 2010.
rural people most benefited. As a result, intergenerational mobility of education was greatly accelerated. Following the period of economic reform and opening (1978 onwards), however, the trend of improvement has reversed. Despite the government’s efforts to enforce nine-year compulsory education for all children, the average schooling for rural youth between 20 and 29 years of age in 2010 was barely above eight years. In contrast, the average educational achievement of the same cohort in urban centres was close to 11 years. The figures for higher education are even worse. In their influential work (Liang et al. 2012), James Lee and his co-authors show that Peking University and Suzhou University, two of China’s elite universities, had a ‘silent revolution’ that extended admission to lower class young people in the period 1949–2002. This experience may not, however, represent the whole country. While China entered a period of expansion of higher education at the turn of the century, it is unclear whether young rural people, who are disadvantaged by the increasing costs of higher education and their families’ need for them to enter the labour market, have benefited from this expansion. The resulting urban–rural divide is the single-most important factor responsible for the stagnation in China’s transmission of intergenerational education. The 70-year history of the People’s Republic was an experiment in social transformation. In its first 30 years, equality was enhanced by the CCP’s efforts of social and political transformation; the following 40 years have witnessed a return to inequality due to a weakening in the CCP’s commitment to a socially and politically equal society.

This paper, using data provided by the China Family Panel Studies (CFPS), documents the trends of improvements and intergenerational mobility in education for people born between 1930 and 1985. While a full consideration of these trends needs a comprehensive study, this paper provides some tentative – in many cases conjectural – explanations and prescribes policy remedies to improve intergenerational mobility in China.

The general trends

The CFPS has had four waves of survey since 2010, but data from the latest wave (2016) were not released in time for this paper’s preparation. This study refers to data from the 2010 survey and, because the paper is not concerned with long-term trends, the 2012 or 2014 surveys are not relevant here.
Improvement of education

The following analysis applies the five CFPS categories of education: 1 = not finishing elementary school, 2 = elementary school, 3 = middle school, 4 = high school, 5 = college or above. To ensure sufficient observations to make reasonable inferences, the sample is restricted to individuals who were 80 or younger in 2010 (i.e. those born in 1930 or later). In addition, to ensure the study considers only people who had completed their education, individuals younger than 25 in 2010 (i.e. those born later than 1985) are excluded. The resulting sample size is 62,219 people who were born between 1930 and 1985.

Figure 7.1. Education trend: Full sample
Source. CFPS 2010

Figure 7.1 presents the composition of educational achievements by birth year. A clear trend of improved education is evident in the 55 years covered by the figure. Among people born in 1930, close to 70 per cent did not finish elementary school; in contrast, this percentage dropped to 7.3 per cent for people born in 1985. The establishment of the New China in 1949 did not accelerate the progress and, on the other hand, a mild setback happened in the later stage of the Cultural Revolution. People born around 1965 should have received elementary education in the early 1970s, but fewer people among them finished elementary
school than those who were born in the early 1960s. The New China did, however, accelerate the spread of middle school education; people born after 1940, who were supposed to obtain middle school education after 1949, were more likely to finish middle school than the earlier cohorts. The acceleration of high school education, though, had to wait until the 1960s. As the figure shows, the chance that people born after 1950 would finish high school increased at that time. This improvement culminated with the cohorts born in the early 1960s who received high school education in the early 1970s. This was consistent with the observation that high school education was greatly expanded during the Cultural Revolution. The quality of high school education at that time, however, was limited because teachers were required to impart to students practical skills instead of general knowledge. This is confirmed by CFPS 2010 data applied to the rate of return to education. People around 45 years old in 2010 (born around 1965) had the lowest rates of return to education among all the income earners in the sample (Yao & Cui 2015). After the death of Chairman Mao Zedong in 1976, the zeal of the Cultural Revolution receded quickly. The share of high school graduates has since stabilised at around 13 per cent.

In the early times of New China (the 1950s and early 1960s), there was a modest expansion of higher education. Among people born between 1935 and 1942 (who reached college age in the 1950s and early 1960s), 4.9 per cent had a college degree, compared with 4 per cent of people born before them. The expansion ceased, however, for the following 20 years when the chance that people born between 1945 and 1960 would attend college dropped to 3.8 per cent. The Cultural Revolution was clearly the cause for this setback because, during its first years, university admission was effectively stopped and, in its later years, only a small number of students were admitted (Liang et al. 2012). The expansion of higher education began again in 1978 when the birth cohorts of the early 1960s reached college age. The share of college graduates reached 21.2 per cent for the 1985 cohort.
The trends presented in Figure 7.2 are broadly similar, although differences are evident for different cohorts. A significantly higher ratio of women born before the early 1960s did not finish elementary school, compared to their male counterparts. The expansion of higher education before the Cultural Revolution benefited more men than women. So did
the expansion of high school education during the Cultural Revolution. In general, however, gender gaps declined over the period and this was driven by female political participation. Using data provided by county chronicles and the 1990 Census, Yao and You (2018) found that, in counties with a faster expansion of female membership in the CCP, the female–male ratio of educational achievement increased more rapidly. Using data from CFPS 2010, they also found that living as an adolescent in a county with larger female party membership induced a person to invest more in his/her daughters.

The rapid decline in gender gaps in the 1980s was compounded by many factors, such as higher returns to education, smaller families, higher family income and continually changing social norms. By the 1985 cohort, the only significant remaining gender gap was at the level of college education. While 23.9 per cent of men born in 1985 had a college degree, the percentage for women born in the same year was 18.6 per cent.

**Intergenerational transmission of education**

The most commonly used indicator for intergenerational transmission of education is the coefficient of correlation between the educational achievements of two consecutive generations. We use this indicator in our paper. Subsequently, we will call it the ‘transmission coefficient’. It is obtained by regressing the child’s education on the higher achievement of his/her parents’ educational achievements. A higher coefficient implies more stagnant intergenerational mobility and thus slower improvement of educational equality.

Figure 7.3 presents the transmission coefficients through a five-year moving cohort. The horizontal axis indicates the starting year of each five-year birth cohort. The figure conveys the main message of this paper: history is repeating itself. The transmission coefficients form a U curve with the lowest points situated at the birth cohorts of the mid-1950s. Intergenerational mobility was less dynamic for people born in the early 1930s and their transmission coefficients were around 0.55. The coefficient dropped rapidly, however, until the cohorts born around 1940, which shows that, even before the communist revolution, social mobility was accelerating. After a short set back, the decline continued until the birth cohorts of the mid-1950s when the transmission coefficient reached its lowest level of 0.32. Most of these people received their middle school and high school education during the Cultural Revolution. The political and social mobilisation during this period had a decisive impact on accelerating
social mobility within the Chinese population. Soon after, China waved goodbye to Mao’s radicalism when he died in 1976, however, history has quickly begun to repeat. The transmission coefficient has increased steadily across birth cohorts and, by the time of the cohorts of the mid-1970s (who would finish their education by the end of the 1990s), it went back to the levels of the cohorts of the mid-1930s. Despite dropping for a brief period, the coefficient turned upward again among more recent cohorts.

Figure 7.3. Transmission coefficient: Full sample
Note. 1. Education is measured in units of five
2. Each point is estimated using a five-year moving sample
3. X-axis indicates the starting year of each sample window
Source. CFPS 2010

Figure 7.4 presents an historical pattern of education transition from parents to children. The child sample is divided into three groups by birth cohort: 1930–44, 1945–64, and 1965–85. Most people in the first cohort, if they received any education at all, began their education before 1949. In the second cohort, most finished their elementary education before 1976, the last year of the Cultural Revolution. Most of the last cohort began their education after 1976. Several findings are evident in the figure. First, given parents’ educational level, more recent cohorts had higher levels of education. This fits into the general trend of improved education shown by Figure 7.1; a rising tide lifts all boats. Second, the 1945–64 cohort had the smallest slope of transition (transmission coefficient), while the other two cohorts had almost the same and larger slopes. That is, after
a period of more equitable improvement, intergenerational transmission of education again lost dynamism in more recent generations. Third, the reduced level of transmission for the 1945–64 cohort was achieved by raising the level of education of those born to parents of lower levels of education more than the level of people born to parents of higher levels of education. Lastly, the transmission coefficient of the 1965–85 cohort increased because people born to parents with lower levels of education did not improve much, while people born to parents with higher levels of education had large improvements.

Figure 7.4. Transmission by cohorts

Note. 1 = Not finishing elementary school
2 = Elementary school
3 = Junior middle school
4 = Senior high school
5 = College or above
Source. CFPS 2010

The last finding is alarming because it indicates that inequality of intergenerational transmission has accelerated among more recent generations: children born to less-educated parents have been locked in by stagnant transmission while children born to more educated parents have been able to obtain much better education than their already highly educated parents. If they went to college at all, the members of the 1965–85 cohort started their college education before 2003. The increasing inequality among this cohort is inconsistent with the findings of James
Lee’s team, which may be because they only studied two elite universities, Peking University and Suzhou University, and thus missed the larger picture of the whole country.

As expected, women’s education was more dependent on their parents’ education than men. Figure 7.5 shows that women had larger transmission coefficients than men, except for people born in the early 1930s. One possible explanation is that women born in that period of time had uniformly low education regardless of their family backgrounds.

![Figure 7.5. Transmission coefficient by gender](image)

**Figure 7.5. Transmission coefficient by gender**

Note. 1. Education is measured in units of five
2. Each point is estimated using a five-year moving sample
3. X-axis indicates the starting year of each sample window
Source. CFPS 2010

How does China’s experience compare with other countries? Hertz et al. (2008) find that the global average correlation between parents’ and children’s schooling held steady at about 0.4 for the 50 years before the global financial crisis (GFC) of 2007–08. Latin America had the highest transmission coefficient of 0.83, followed by Asia (0.69), sub-Saharan Africa (0.66), and Western Europe and the United States (0.52). The Eastern Bloc fared the best, having a transmission coefficient of 0.38. Large variations are evident within regions; for example, Azomahou and Yitbarek (2016) find that, among several sub-Saharan countries, Nigeria had the highest level of education inheritance, with its transmission coefficient reaching above 1.2 for people born around 1950, whereas
Ghana fared much better, with its transmission coefficient declining from 0.77 of the birth cohort of 1944–48 to 0.39 of the birth cohort of 1989–93. These findings, compared with this paper’s findings about China, reveal that China did not fare badly even before 1949. The only region that outperformed China was the Eastern Bloc, which experienced communist revolutions. Similar to the Eastern Bloc, China’s communist revolution also had a significant levelling effect, so that, by the time of the Cultural Revolution, education transmission declined to the same level. In the 40 years of the reform era, China has returned to the level of Western Europe and the United States, which is above the world average. As in many other social arenas, China resembles a typical country of market economy.

Rural–urban disparities

One of the most significant forms of inequality in China is the rural–urban divide. Urban per capita income reached 3.3 times that of rural per capita income before the GFC. There has been substantial decline in more recent years, but it was still 2.7 times greater in 2016. The educational gap between the city and countryside is also large. In breaking up the data from Figure 7.1 into urban and rural sub-samples, Figure 7.6 reveals a sharp contrast. The divide started with people born in the 1930s, but was narrowed for people born in the 1950s and early 1960s. Educational expansion in the Cultural Revolution lifted the educational achievements in cities and the countryside, but had a larger impact on the countryside. Divergence began, however, soon after college admission was resumed in 1977. More and more urban youths began to attend college and, when the 1985 birth cohort reached the age of higher education, more than half of urban youths attended college. In contrast, the figure was barely above 10 per cent in the countryside. Yet the divergence did not stop at the higher end of education. Among the 1985 cohort, there was almost no urban resident who did not finish elementary school, but nearly 10 per cent of rural residents did not. Close to 80 per cent of the rural residents in the 1985 cohort did not have high school or higher levels of education, while 75 per cent of their urban counterparts did.

2 This contrast may be exaggerated, though, because many rural youths who have a college education stay in the city. The results would be neater if individuals could be categorised by their parents’ Hukou (household registration system that identifies an individual as a rural or urban resident). Unfortunately, CFPS does not request this information.
Figure 7.6. Education trend by Hukou

Source. CFPS 2010
Intergenerational transmission may have enlarged the rural–urban divide. Figure 7.7 presents the rural and urban transmission coefficients against the national coefficients. In the birth cohorts of the 1930s, urban and rural transmission coefficients were about the same size. For people born in the 1940s, however, the countryside had smaller coefficients than the city. Literacy programs were a central component of the CCP’s modernisation drive, even before it controlled the whole country in 1949. Those programs had more significant effects in the countryside, and the birth cohorts of the 1940s benefited the most from them. As a result, their educational achievements diverged from their parents. Starting from the birth cohorts of the early 1950s, the transmission coefficient began to increase in both the countryside and the city, and the gap between the countryside and the city was not large. There is, however, one period that is worthy of attention. While the coefficient was generally larger in the countryside than in the city, the countryside had a smaller coefficient than the city for people born in the 1960s. Clearly, the broad effect of educational expansion in the Cultural Revolution was to break up intergenerational transmission in the countryside more than in the city, mostly because the countryside started from a much lower basis than the city.

**Figure 7.7. Transmission coefficient by Hukou**

*Note.* 1. Education is measured in units of five
2. Each point is estimated using a five-year moving sample
3. X-axis indicates the starting year of each sample window

*Source.* CFPS 2010
Figure 7.7 also shows that the national transmission coefficients were much larger than those of either the countryside or the city. Because the gap between the city and the countryside was not large in most time periods, we have reason to believe that the higher national coefficients were a result of between-group variations of educational level. Figure 7.8 illustrates the idea. In the figure, the horizontal axis represents the highest educational achievement of parents ($E_{t-1}$), and the vertical axis is the highest educational achievement of children ($E_t$). The two shaded ovals indicate the urban and rural samples, respectively. To reflect the reality that urban educational levels were higher than rural educational levels, the urban sample is drawn to lie above the rural sample. The two dashed lines are the respective regression lines for the two samples. The slopes, which are the transmission coefficients for the city and the countryside, are not markedly different. The national transmission coefficient is the slope of the regression line for the two samples combined. Because the urban sample lies above the rural sample, the national regression line is steeper than either the rural or the urban regression line. In the Appendix, we develop a decomposition method to separate the national transmission coefficient into three parts: the contribution of urban transmission coefficient, the contribution of rural transmission coefficient, and the contribution
of urban–rural educational gaps. The size of the first two components depends on the size of the respective transmission coefficient and the share of the sum of variations of the correlation between parents’ and children’s education in the urban and rural samples, and the size of the third component depends on the urban–rural gap of education and the urban–rural gap of transmission coefficient weighted by the share of the sum of variations of the correlation between parents’ and children’s education.

**Figure 7.9. Decomposition of transmission coefficient**

Note. 1. Education is measured in units of five  
2. Each point is estimated using a five-year moving sample  
3. X-axis indicates the starting year of each sample window  
Source. CFPS 2010

Figure 7.9 presents the shares of contribution of the three components and shows three regularities. First, the contribution of the urban transmission coefficient declined over time, particularly since the cohort of 1945. Because the urban transmission coefficient increased for the more recent cohorts, this result is caused by the shrinking share of variations in the correlation between urban parents’ and urban children’s educational achievements in the national total variations. Second, the contribution of the rural transmission coefficient increased substantially since the 1950s. For the more recent cohorts, it became the largest contributor. Because the rural transmission coefficient was not substantially larger than the urban transmission coefficient, its larger contribution was created by its larger share of the variations of the correlation between parents’ and children’s
educational achievements. Third, the contribution of the urban–rural gap of education experienced a U–shaped curve, with its lowest points located between the 1950 cohort and the 1965 cohort. This is a surprising result because the urban–rural divide declined the quickest for those cohorts. One explanation is that improvements in urban education were more dynamic and diverse than in rural education in this period of time, so the urban sample contributes a larger share of variation to the estimation. It is likely that the decline in the contribution of the urban–rural gap in more recent history probably was caused by the shrinking gap between the urban and rural transmission coefficients.

Conclusion

The twentieth century witnessed dramatic changes in Chinese society. Ending a long history of stagnation, the start of the century saw China begin a journey of social transformation that was greatly accelerated by the communist revolution. The first 30 years of the People's Republic marked Chinese society with equality. Most significantly, the rural educational level was dramatically raised and, as a result, intergenerational educational mobility increased. This process has reversed since 1978, when the CCP turned its focus to economic growth and spent less effort on social transformation. Improvements in rural education have been left behind and educational achievements no longer keep pace with urban areas. Consequently, intergenerational mobility has been dragged back.

Although the Chinese Government has increased fiscal spending on nine years of compulsory education, investment in high school and college education has been left largely to families. In the countryside, several reasons have contributed to a decline in families' investment in their children's education. First, the high demand for labour in coastal cities has raised the opportunity cost of education. With a middle school diploma, a rural youth can easily find a reasonably paid job at the coast. Second, good high schools are more concentrated in the city and out of the reach of rural youths. As a result, rural youths often end up attending third-tier universities, even if they finish high school. Third, China has a regressive tuition system in its higher education sector and lower quality universities charge higher tuition than better universities (e.g. the tuition of Peking University, which is 5,000 yuan per academic year, has not changed
over the last 20 years, but some private universities in Beijing charge 40,000 yuan for an academic year). Rural youths thus have to face two disadvantages: poor quality university education and high tuition cost.

To prevent history from repeating itself, China must reintroduce some of the progressive programs that the government abandoned over the last 40 years. New measures include, but are not limited to, allocating more educational resources to the rural areas (including teacher subsidies), extending compulsory education to 12 years, and allowing elite universities to charge higher tuitions and sparing government finance to support ordinary universities/colleges. This requires that the government put social progress as one of its priorities. During 40 years of relentless economic growth, the Chinese Government and Chinese society have been pre-occupied by a single-minded belief in economic efficiency. Now that China is moving towards a more affluent society, it is time for the government to reintroduce social progress into its programs.

References


Appendix

The regression is as follows:

\[ y_i = \alpha + \beta x_i + u_i \]

So the OLS estimator is formulated as

\[
\hat{\beta} = \frac{\sum_i (x_i - \bar{x}) y_i}{\sum_i (x_i - \bar{x})^2} \approx \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} + \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

We further denote the means of \( x \) in urban and rural samples as \( x_u \) and \( x_r \), and let \( x_u = x_r + \delta \), where \( \delta \) represents the urban–rural education gap. Meanwhile, we denote the total/urban/rural number of observations as \( N/N_u/N_r \). So we have

\[
\bar{x} = \bar{x}_u \frac{N_u}{N} + \bar{x}_r \frac{N_r}{N} = \bar{x}_u - \delta \frac{N_r}{N} = \bar{x}_r + \delta \frac{N_u}{N}
\]

So

\[
\hat{\beta} = \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} + \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} + \hat{\beta}_r \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2}
\]

\[
= \hat{\beta}_u [1 + \delta \frac{N_r}{N}] \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} + \hat{\beta}_r [1 - \delta \frac{N_u}{N}] \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]

\[
= \hat{\beta}_u \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}_u) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x}_u)^2} \frac{\sum_{i \in \text{urban}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{urban}} (x_i - \bar{x})^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2} \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}) y_i}{\sum_{i \in \text{rural}} (x_i - \bar{x})^2}
\]
Rearranging the equation, we shall have

\[
\hat{\beta} = \hat{\beta}_u \frac{SST_{x,u}}{SST_x} + \hat{\beta}_r \frac{SST_{x,r}}{SST_x} \\
+ \delta \left[ \hat{\beta}_u \frac{\sum_{i \in \text{urban}} y_i}{N} \frac{SST_{x,u}}{SST_x} - \hat{\beta}_r \frac{\sum_{i \in \text{rural}} y_i}{N} \frac{SST_{x,r}}{SST_x} \right]
\]

where \( \frac{SST_{x,r}}{SST_x} = \frac{\sum_{i \in \text{rural}} (x_i - \bar{x}_r)^2}{\sum_i (x_i - \bar{x})^2} \). And in the graphs, we denote \( \hat{\beta}_u \frac{SST_{x,u}}{SST_x} \) as the urban component, \( \hat{\beta}_r \frac{SST_{x,r}}{SST_x} \) as the rural component and the remaining as the urban–rural gap component.