INFORMATIVE PSYCHOMETRIC FILTERS

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Previous Books by the Author

Psychometrics of Similarity
Time Series in Psychology
Nonlinear Psychophysical Dynamics
n-Dimensional Nonlinear Psychophysics
Cascades and Fields in Perceptual Psychophysics
Preface

There has been a time gap in what I have published in monograph form, during which some very important and profound shifts have developed in the way we can represent and analyse the sorts of data that human organisms generate as they progress through their lives. In that gap I have been fortunate enough to make contact with and exchange ideas with some people who have chosen to explore nonlinear dynamics, and to see if the ideas that are either pure mathematics or applications in various disciplines other than psychology can be usefully explored as filters of behavioural time series.

Three strands in my previous published work have come together here; they are time series, psychophysics, and nonlinear nearly chaotic dynamics. Also, the width in scale of the data examples examined has broadened to include some psychophysiological and social processes. There has evolved a proliferation of indices, hopefully to identify what is happening in behavioural data, that can be regarded as partial filters of information with very varying efficiency.

The title of this effort was selected to emphasise that what is measured is often information or entropy, and that the focus is on problems and data that are ill-behaved as compared with what might be found in, say, physics or engineering or neurophysiology. The object is to model in a selective way, to bring out some features of an underlying process that make some sense, and to avoid misidentifying signal as noise or noise as signal.

Recent developments in psychophysiology (Friston, 2005) have employed networks of mixed forward, reverse and lateral processes, some
of which are linear and some nonlinear. That form of construction brings us closer to neurophysiological cortical structures, and takes theory further than is pursued here, though there is in both approaches an explicit assumption of nonlinear mappings playing a central role in what is now called the inverse problem; that is to say, working back from input-output data to the identification of a most-probable generating process.

There is no general and necessary relationship between identifiability, predictability and controllability in processes that we seek to understand as they evolve through time. In the physical sciences sometimes the three are sufficiently linked that we can model, and from a good fitting model we can predict and control. But in many areas we may be able to control without more than very local prediction, or predict without controlling, because the process under study is simple and linear, and autonomous from environmental perturbations. That does not hold in the life sciences, particularly in psychology outside the psychophysical laboratory.

The unidentifiability or undecidability in identification, prediction and control can be expressed in information measures, and in turn, using symbolic dynamics, can be expressed theoretically in terms of trajectories of attractors on manifolds. It is the extension of our ideas from linear autocorrelation and regression models to nonlinear dynamics that has belatedly impacted on some areas of psychology explored here.

A related problem that is unresolved in the current literature, for example in various insightful studies in the journal *Neural Computation*, is that of defining complexity. The precise and identifiable differences between complexity and randomness have been a stumbling block for those who want to advance very general metrics for differentiating the entropy properties of some real data in time-series form. I have not added to that dispute here, but sought simply to illustrate what sorts of complicated, non-stationary and locally unpredictable behaviour are ubiquitous in some areas of psychology. The approach is more akin to exploratory data analysis than to an algebraic formalism, without wishing to disparage either.

The problems of distinguishing between the trajectories of deterministic processes and the sequential outputs of stochastic processes, and consequently the related problem of identifying the component dynamics of
mixtures of the two types of evolution, has produced a very extensive literature of theory and methods. One method that frequently features is so-called box-counting or cell-mapping, where a closed trajectory is trapped in a series of small contiguous regions as a precursor to computing measures of the dynamics, particularly the fractal dimensionality (for an example, which has parallels in the analysis of cubic maps in nonlinear psychophysics, see Udvdia and Guttalu, 1989).

Serious difficulties are met in identifying underlying dynamical processes when real data series are relatively short and the stochastic part is treated as noise (Aguirre & Billings, 1995), it is not necessarily the case that treating noise as additive and linearly superimposed is generically valid (Bethet, Petrossian, Residori, Roman & Fauve, 2003). Though diverse methods are successfully in use in analysing the typical data of some disciplines, as in engineering, there are still apparently irresolvable intractabilities in exploring the biological sciences (particularly including psychology), and a proliferation of tentative modifications and computational devices have thus been proposed in the current literature.

The theoretical literature is dominated by examples from physics, such as considerations of quantum chaos, which are not demonstrably relevant for our purposes here. Special models are also created in economics, but macroeconomics is theoretically far removed from most viable models in psychophysiology. Models of individual choice, and the microeconomics of investor decisions, may have some interest for cognitive science, but the latter appears to be more fashionably grounded, at present, in neural networks, though again the problem of simultaneous small sample sizes, nonlinearity, non-stationarity, and high noise have been recognised and addressed (Lawrence, Tsoi & Giles, 1996).

One other important social change in the way sciences exchange information has in the last decade almost overtaken the printed word. For any one reference that can be cited in hard copy, a score or more are immediately identifiable in internet sources such as Google, and the changes and extensions of ideas, and perhaps also their refutation, happens at a rate that bypasses the printed text even under revisions and new editions. For this reason, there are some important topics that are not covered here,
tools such as Jensen-Shannon divergence are related to entropy and to
metric information and could well be used to augment the treatment of
nonlinear and non-stationary psychological data but so far have not been
considered. We urge the reader to augment and criticise the present text
by checking developments in the electronic sources, particulary focussing
on work such as that by Fuglede and Topsøe (www.math.ku.dk/topsoe/)
on Jensen-Shannon divergence, or Nicolis and coworkers (2005) on dy-
namical aspects of interaction networks, that have relevance and promise.
Jumps between modes of dynamical evolution even within one time se-
ries essentially characterise psychological processes, and transient states
such as chimera (Abrams and Strogatz, 2006) may yet be identified in psy-
chophysiology.

I want to thank various people who have encouraged or provoked me
to try this filtering approach, and to bring together my more recent work
that is scattered over published and unpublished papers, conference pre-
sentations, invited book chapters, and even in book reviews. One very con-
genial aspect of the modern developments in applied nonlinear dynam-
ics is the conspicuously international character of the activity. Professors
Stephen Guastello and Fred Abraham in the USA, Hannes Eisler in Swe-
den, John Geake in England, Ana Garriga-Trillo in Spain, and Don Byrne
and Rachel Heath in Australia, have all offered me constructive help or
encouragement over the last decade.

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