Appendix 6: Uncertainty, risk and sensitivity

Although his focus is on sensitivity testing of economic models, Pannell (1997) notes the paucity in the literature of 'discussion and procedures and methodological issues for simple approaches to sensitivity analysis'. Even classical cost-benefit analysis (CBA) textbooks such as Harberger (1976), Mishan (1988), Sugden and Williams (1978), and Gramlich (1981) have limited discussion of sensitivity analysis per se, with generally minimal attention paid to uncertainty and risk.

Some government-sponsored manuals conflate the concepts of risk analysis and sensitivity analysis. For example, the Austroads-sponsored project evaluation guide by Rockliffe et al. (2012, p. 24) states that sensitivity analysis 'is the simplest kind of risk analysis', although the following pages do in fact separately discuss quantitative risk analysis in probability terms. More disconcerting is the confounding by Rockliffe et al. (2012, pp. 29, 46–54) of the technical economic concept of risk that is used in CBA risk analysis with the more prosaic concept of a qualitative or quantitative combination of likelihood and negative impact. In everyday language, too, risk is generally perceived to constitute a negative outcome.

While the term 'uncertainty' is generally used in different ways to denote lack of precise knowledge or certainty, 'risk analysis' and 'sensitivity analysis' have specific and distinct meanings in CBA. The distinction is ostensibly based on definitions of risk and uncertainty originally put forward by Knight in 1921 (2009), and which have since become conventional usage among economists. Following Knight, risk is generally conceptualised as positive or negative variation in outcomes from a measure of central tendency, where the probabilities of deviations are known.[1] Uncertainty is associated with unpredictable variation because the probability of events is not known. Recent usage, including in military circles and the climate change literature, may also refer to 'deep uncertainty' (an 'unknown unknown'), which implies lack of knowledge even of the nature of a future event.

A6.1 Sensitivity analysis

Sensitivity analysis is a traditional aspect of project evaluation. Sinden and Thampapillai (1995, ch. 10), devote a chapter to the topic, and define it as follows:

> A sensitivity test is a recalculation of net social benefits with different data, together with the reinterpretation of the relative desirability of the alternatives.

An analyst has varying degrees of confidence in the values used to estimate the net present value of a project. For example, if the purchase of a standard item of equipment that is supplied in a competitive market is to occur soon after commencement of a project, then its price is likely to remain close to the initial estimate. Costs and benefits incurred or reaped further out into the future, on the other hand, are more likely to deviate from current estimates.

In principle, the analyst could assume a number of different values for each variable. In the case of the cost of a standard item of equipment early in the project, it could be assumed that the upper and lower bounds might be 1 per cent either side of the current

1 Knight (2009, p. 121) distinguishes risk and uncertainty in a number of places. However, the most cogent distinction is at the beginning of Chapter 8:

> To preserve the distinction which has been drawn in the last chapter between the measurable uncertainty and an unmeasurable one we may use the term 'risk' to designate the former and the term 'uncertainty' for the latter. The word 'risk' is ordinarily used in a loose way to refer to any sort of uncertainty viewed from the standpoint of the unfavourable contingency, and the term 'uncertainty' similarly with reference to the favourable outcome; we speak of the 'risk' of a loss, the 'uncertainty' of a gain … The practical difference between the two categories, risk and uncertainty, is that in the former the distribution of the outcome in a group of instances is known (either through calculation a priori from statistics of past experience), while in the case of uncertainty this is not true, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique.

estimate. An estimate for a cost 20 years into the future may merit a range that is 15 per cent either side of current expectations. Rockliffe et al. (2012, Table 5.3) suggest ranges for sensitivity analysis such as a plus or minus 50 per cent variation either side of the estimated traffic diverted or generated by a transport project, but only $\pm$ 0.3 for estimated average car occupancy.

Having set the likely range or bounds for each variable, the analyst would then estimate the net present value (NPV) for the lowest (perhaps 'worst case'), highest (perhaps 'best case') and expected ('most likely' or plausible) levels of each variable. However, the number of results increases exponentially with the number of variables tested. If three levels (e.g. high, medium, and low) are used for each variable, then two variables will require $3^2 = 9$ separate calculations. Four variables would require $3^4 = 81$ calculations of NPV. Whether a decision-maker would find it useful to be presented with 81 separate possible values of NPV is open to doubt. Because it does not involve probabilities, sensitivity analysis provides no guidance as to which one of the 81 results is to be preferred.

Because comprehensive application of sensitivity analysis is subject to diminishing returns in a large, complex project, Sinden and Thampapillai (1995, ch. 10) recommend the exercise of judgement by limiting testing to so-called 'critical' variables that are likely to affect the calculated NPV so much that the project may be abandoned, or an alternative project chosen. Nevertheless, they acknowledge that the only reliable method of identifying critical variables is by systematic recalculation of NPVs, a task that is made easier by the use of spreadsheets and computers.

Little and Mirrlees (1974, p. 309) are explicitly sceptical about the usefulness of sensitivity analysis to decision-makers faced with more than one estimate of NPV. Squire and van der Tak (1975, p. 45) point out that different variables may be positively or negatively correlated, but sensitivity analysis assumes that variables are independent of each other (Campbell & Brown, 2003, p. 197). Perkins (1994, s. 15.7.2) considers that a key weakness of sensitivity analysis is its use of 'randomly selected percentage values, such as 10 per cent or 20 per cent', rather than by standardised amounts such as one standard deviation. Finally, selection of 'best' and 'worst' case limits for a variable implies the selection of low-probability events that may be highly unlikely to occur, but are likely to alter the calculated NPV noticeably, bringing into question the purpose of any sensitivity analysis conducted on the basis of extreme values.

Despite these complications, sensitivity analysis can play a useful role in CBA. If a small change in a particular variable produces a large change in NPV, then a prudent analyst will check the method and data used to estimate that particular variable. If appropriate, some re-estimation may be justified. Even if there is sufficient confidence in the robustness of the estimated value, there may be a case for applying risk analysis to the evaluation of the project.

An alternative application of sensitivity analysis is to identify the value of a variable, or group of variables, where NPV falls to zero; the so-called 'switching', 'cross over', or 'break-even' value. Knowledge of switching points can assist a decision-maker to assess the plausibility of the evaluation results.

Of particular relevance to possible harmonisation of the approach to conducting sensitivity analysis is the identification by Sinden and Thampapillai (1995, ch. 10) of potential misuses of the technique. They recommend that analysts should:

- summarise in the main report break-even values of variables, leaving the presentation of large numbers of NPVs from recalculations based on 'several levels of several variables' to an appendix
- provide an interpretation of the results of sensitivity analysis, avoiding just the presentation of arithmetic results of such things as recalculations of NPVs or break-even values
- integrate sensitivity tests into the overall analysis by demonstrating which variables are critical to making choices between the project at hand and any alternatives
- not use sensitivity tests as a substitute for the valuation of unpriced outcomes.

It is important to note that sensitivity analysis is carried out without involving the use or application of probabilities. Risk analysis, on the other hand, is based explicitly on the application of probabilities. The specification of probabilities, or probability distributions, however, is also subject to uncertainty (in the everyday usage of the term). It is therefore appropriate that sensitivity analysis be applied even to the results of risk analysis — a topic that is addressed below.

A6.2 Risk analysis

Risk analysis can be carried out using expected values (probability weighted values of different variables) or with decision trees: Boardman et al. (2011, ch. 7) provide a detailed exposition. The availability of modern software facilitates the application of Monte Carlo analysis. Campbell and Brown (2003, ch. 9) provide an accessible explanation of Monte Carlo analysis using snapshots of spreadsheets based on the @RISK software (www.palisade.com).

In essence, risk analysis requires the specification of probability distributions for some or all of the variables utilised in a project evaluation. Section A6.1 on sensitivity analysis cited Rockliffe et al. (2012, table 5.3) suggesting a 50 per cent variation either side of the estimated traffic diverted or generated by a transport project. The traffic volume variable could be represented by a symmetrical triangular distribution with its peak at the most likely or expected value of, say, 100,000 vehicles per period, and the range ±50 per cent either side of this value, as illustrated in Figure A6.1. Note that sensitivity analysis would only have utilised the three values shown on the horizontal axis, but risk analysis further specifies the probability of observing those three values, as well as the intermediate ones.

@RISK provides a menu of different probability distributions (e.g. normal, log-normal, binomial, Poisson) that can be specified as appropriate for each variable in an evaluation. The program then draws values randomly from each of the specified distributions and calculates NPV by combining the randomly selected values. The first draw may produce an NPV = x_1. The second draw may result in NPV = x_2, the third a different NPV = x_3, and so on. A large number of draws (e.g. 10,000) will produce a histogram of NPV values. The histogram, or its smoothed probability distribution can be used to determine the probability of achieving any particular NPV value.

In other words, the representation of variable values by probability distributions rather than point estimates permits the analyst to incorporate risk directly into the calculation of NPV. The advantage of the Monte Carlo method is its combining of the risks attached to all variables into a single distributional outcome. It also has the advantage of avoiding more dubious approaches to allowing for risk, like adding a risk premium to the discount rate. But a particular disadvantage of the Monte Carlo method that cannot be easily or totally overcome is the treatment of correlated variables.

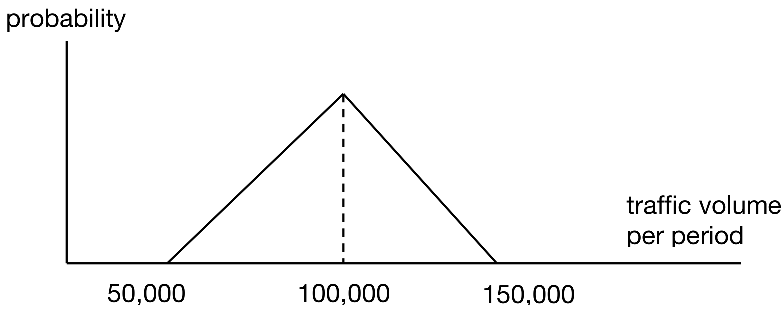

Figure A6.1: Illustration of use of triangular distribution to represent estimated traffic volumes in a transport project evaluation

Source: Leo Dobes

A6.2.1 Sensitivity testing of a risk analysis result

Sensitivity testing need not be limited to non-probabilistic point estimates of variable values. It can, and should, also be applied to the results of risk analysis.

The triangular distribution shown in Figure A6.1, for example, can be subjected to sensitivity analysis. The triangular distribution is defined by two parameters: the extremes of its range (50,000 and 150,000) and the most likely value (100,000). Standard sensitivity testing of point estimates might have investigated the effect of changing the value of the traffic volume variable from the point estimate by also calculating NPV for the two values at either end of the range. The decision-maker would have been presented with three results, rather than one NPV value.

There is no reason, however, why risk should be represented by a single, static probability distribution. An analyst might just as well have set the range of traffic volumes at, say, from 40,000 to 160,000, forming a different symmetrical triangular distribution with a greater variance. Selection of a value range like 30,000 to 110,000 would generate a skewed triangular distribution.

By specifying a set of values starting, for example, at 30,000 and increasing by increments of 10,000 up to 170,000, it is possible to allow @RISK to select randomly from a range of different triangular probability distributions for the variable 'traffic volume per period'. Sensitivity analysis can now be carried out on changes in the most likely value of 100,000, as well as on its associated probability distributions. In principle, the sensitivity analysis of Monte Carlo results will incorporate the first three moments of the distribution — mean, variance and skewness — when testing the sensitivity of calculated NPVs.

A6.2.2 Discount rates and sensitivity analysis

Australian practice is typically to vary the discount rate as part of sensitivity tests. Infrastructure Australia (2013, p. 9) requires the use of 4 per cent, 7 per cent, and 10 per cent per annum in real terms for submission of project proposals, and justifies these values on the basis of common usage in the majority of Australian jurisdictions. The 4 per cent and 10 per cent real per annum rates are based on the assumed social rate of time preference and the social opportunity cost of capital approaches respectively. Nevertheless, specification of multiple discount rates for use in sensitivity analysis would merit extensive deliberation in harmonisation of approaches to CBA.

Market rates of interest can encompass a range of values, depending on the nature of various specialised financial markets. They can also vary with short-term market conditions. On this basis, one could surmise that sensitivity analysis applied to financial or investment analysis would include more than one value for the discount rate. The section on sensitivity analysis in a key text on corporate finance by Brealey et al. (2006, pp. 245–56), however, focuses on a range of project variables, but does not refer at all to varying the discount rate.

Most textbooks on CBA, on the other hand, include some reference to varying discount rates as part of sensitivity analysis. Yet the same CBA textbooks also devote space to discussing an appropriate social discount rate, or some average or shadow value, implying that society has a specific or particular time preference rate, absent complications like tax wedges, expression of costs and benefits in consumption or investment equivalents, and intergenerational issues. Conceptually, a social discount rate is presented implicitly as a point estimate parameter rather than a variable.

Accounting for risk by adding a risk premium to a risk-free discount rate in CBA would require project-specific rates that justify sensitivity testing different rates, but adding risk premiums is rarely recommended in CBA textbooks for reasons summarised by Harrison (2009, ch. 4). Alterations to discount rates may also raise concerns about potential resort to 'fudge factors' in calculating the net social benefit of a project.

Variation of the discount rate in a sensitivity analysis is unlikely to add value or knowledge in the evaluation of a project. The present value formula guarantees that changing the discount rate will result in some change in the calculated value of net social benefit, commensurate with the extent of the variation in discount rate. There is no objective or unambiguous method for choosing the extent of the variation, however, any more than there is for choosing a 'correct' discount rate. And if the discount rate is subjected to sensitivity analysis and found to be influential in determining the NPV, it is not clear how a decision-maker can or should choose between the different results, particularly if the calculated NPVs differ in sign.

Government CBA manuals and textbooks rarely, if ever, provide an explicit justification for sensitivity testing of discount rates. Harrison (2010, p. 61), who supports the application of sensitivity analysis to the discount rate, argues that:

> If the sensitivity analysis reveals that the choice of discount rate is important (changes the sign of the project's net present value), then more consideration should be given to the choice of an appropriate rate — such as the risk characteristics of the proposal (for example, the extent of fixed costs and how costs and benefits vary with the

> state of the economy). Project flows that are more sensitive to market returns and other factors should have a higher discount rate, while projects that are less sensitive should have a lower one.

There are two significant problems with this justification. First, it is not clear how risk premiums are to be determined for public sector projects on the basis of their sensitivity to market returns. The problem is obvious for non-marketed goods like health and education, at least in terms of the fact that government provision of such services differs substantially from limited market counterparts. Private hospitals and schools, for example, may be run on non-profit lines or be held by private owners with no exposure to equity markets.

The second problem with Harrison's (2010) justification is that it is circular. Having presumably chosen an appropriate discount rate, the decision-maker may find that sensitivity testing with different rates yields NPVs of different signs. The recommended solution is to give further consideration to the 'appropriate' rate, an approach that might be interpreted by a sceptic as fiddling with the discount rate to get the desired answer. It further begs the question of the basis for the initial choice of sensitivity values of 4 per cent and 10 per cent per annum. As Harrison (2010, p. 61, fn. 16) correctly points out, the increasingly conventional use of discount rates of 4, 7, 10 per cent per annum appears to represent equal, symmetrical deviations from the 'central' rate of 7 per cent per annum, but in fact produces non-symmetrical effects in present value terms.

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